

Cognitive, Meta-Cognitive, Affective, Social and Behavioral Aspects of Mobile Mathematics Learning

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Abstract

This study is a review of different studies which examined students' learning in the mobile phone environment. Some of these researches treated students' learning in general, while others treated specific aspects of students' learning, like students' building of mathematical knowledge and students' emotions during the learning of mathematics. This study wants to review the already published studies regarding five aspects of students' learning of mathematics in the mobile phone environment: the cognitive, the meta-cognitive, the affective, the social, and the behavioral aspects. This will give us a general picture of mathematics education in the mobile phone environment. The review was done through attempting to characterize the different aspects of middle school students' learning of mathematics in the mobile phone environment, in addition to examining the factors that influenced these different aspects. Further, an attempt will be done to analyze the five aspects as a system that includes the interactions among them.

1. Introduction

To understand student's learning several aspects of this learning should be considered, especially the cognitive [1], behavioural [2], affective [3] and social aspects [4]. In addition, another aspect is being attended to more and more in recent years, namely the meta-cognitive aspect [5]. Analysing these aspects would give us a detailed picture and deep understanding of any learning phenomenon, where in this study we want to do that for the middle school learning of mathematics with the mobile phone. This analysis is especially true when we analyse learning with new technology like the mobile phone, for it will enlighten our vision regarding the potentialities and possibilities of this technology as an educational too.

Analysing students' learning, the cognitive aspect of learning is the most attended to, where it is pointed at, in terms of cognitive outcome, as an important aspect in most learning contexts [1]. When looking at fostering and improving individual as well as group learning, the meta-cognitive aspect is pointed at as most important. Specifically, the development of meta-cognitive expertise is emphasized as crucial for this fostering and improvement [5]. The affect aspect is considered for various reasons, especially its relation with other aspects of students' learning, for example their scholastic development, where [3] concludes, based on theoretical considerations and evidence, that emotions in academic settings are critical to college students' scholastic development. Building a community of learners is attended to increasingly in recent years, where the social presence is considered necessary for building and sustaining these communities [4]. Moreover, as learners become central players, they begin to build social presence, so they enter into dialogue with the community and enhance it [4]. The community's support in turn encourages the learner to further his/her learning and participation. Social presence is also reported to increase the learner's satisfaction in online learning [6].

Students' and teacher's behaviour in the classroom is a fifth aspect of learning which is connected to the previous four aspects and which influences the outcomes of learning. This paper will consider and analyses these five aspects of learning in the context of middle school students' learning of mathematics using the mobile phone. This will be done by analyzing eight studies published in the last three years, where some were written by the author and others by the author in collaboration with Baya'a. Thus, the main goal of this paper is to examine the five aspects of middle school students' learning of math in using the mobile phone, where this learning opens up fascinating opportunities for learning because of its mobility, availability and flexibility [7]. Specifically, the mobility of the mobile phone adds more opportunities to students' learning outside the classroom through carrying out authentic activities. In addition, the technological tools in the mobile phone (the midlets, the time stopper, etc.) support this carrying out of authentic activities. This research examines the different aspects of students' learning of mathematics especially when they carry out such activities while utilizing the mobile phone features.

2. Theoretical background

Using the mobile phone in teaching and learning mathematics in the classroom and especially outdoors developed a learning environment in the sense of [8], where the learning environment has three attributes (ibid, p. 256): (1) The environment is comprised of numerous components, here we are interested in one group of components: the cognitive, the meta-cognitive, the affective, the social and the behavioral; (2) The components interact with each other, affecting each other and giving meaning to each other; and (3) the interactions among the components and their consequences are in constant flux, which makes the learning environment dynamic and not an easily predicted social entity. This sense of the learning environment makes us follow the systematic approach for analyzing the mobile mathematics learning environment ([8], [9]). Doing so, it will be taken care of the various interactions between the components of the learning environment in which this research is interested (primarily in the discussion section), but it will also be taken care of every component in the group, i.e. following the analytic approach described in the two sources above. This will be done in order to take care of precision capitalized in the analytic approach and authenticity capitalized in the systematic approach [9]. The analytic approach will give us a picture of the characteristics of every one of the students' learning responses when they learn mathematics in the mobile phone environment, in addition to the factors that influence the responses in each component of students' learning, while the systematic approach will help show the interactions and the reciprocal influence of the components.

3. Literature review

Several researchers indicated the importance of the cognitive, meta-cognitive, affective, social and behavioral aspects of learning when coming to analyze students' learning and its related phenomena. [10] studied the cognitive, affective and behavior aspects as parts of a theoretical framework to study students' engagement in school. Part of the Rochester Assessment Package for Schools ([11], as reported by [12]) also measures cognitive, behavioural and emotional engagement. [13] added to the three aspects a fourth one: the academic aspect – amount of time engaged completing an academically relevant task. This research will analyze the three first aspects of students' learning of mathematics when using mobile phones (cognitive, affective and behavioural aspects), in addition to the meta-cognitive aspect and the social aspects.

Some researchers related the cognitive aspect to knowledge construction ([14], [15], [16]) and to communities of practice [17]. [17], for example, described how cognitive processes are mediated in communities of practice. The authors reported on eliciting high-level mathematics symbolizing and communicating from students engaged in mathematics communities of practice.

On the other hand, [18] described how sustained and progressive mathematical knowledge building can be fostered in CSCL communities.

The social aspect of learning is also related to mathematics communities. Researchers also related it to students' interactions [19]. [20] depends on Bandura to say that through interacting with peers and the situated environment, individuals' cognition, affection and behavior are influenced.

The meta-cognitive aspect was researched together with the cognitive one, especially the influence of the first on the second. Specifically, researchers refer to meta-cognitive processes as processes related to one's own thinking or processes associated with managing one's own learning. [21], as reported in [22], defines meta-cognition as the knowledge about one's own cognitive processes and products or anything related to them, for example the monitoring, regulation and orchestration of the learning processes related to cognitive objects or data. Further, [14] refers to meta-cognitive students' processes as those of planning, monitoring and evaluating, while [22] refers to them, in the context of mathematical problem solving, as the managerial processes associated with problem solving: understanding the problem, making a plan, carrying out the plan and looking back.

The affective aspect was researched in relation to the cognitive and social aspects [23]. [24] refers to it as a broad range of feelings and moods which are generally considered to be different from pure cognition, including, as specific components, attitudes, beliefs, and emotions. This article will take care of two affective components: students' perceptions of their learning and students' emotions while learning mathematics with the mobile phone.

The behavioural aspect was researched in relation to the cognitive one [25], as well as students learning actions [26] and interactions [2]. In the current research, this aspect will be described in the light of students' actions and interactions enabled when using the mobile phone to learn mathematical topics.

4. Research rationale and goals

Summarizing the above brief and probably partial survey on the five aspects of learning, it could be claimed that these aspects are complementary, and that examining the usefulness of a learning environment entails examining the five aspects, so this article is interested in verifying the characteristics of these five aspects in the mobile phone environment when students participate in outdoor activities to learn mathematics. The current article verifies the characteristics of the five aspects by reviewing different published studies treating learning mathematics with the mobile phone, and which were written partially by the author and partially by the author and Baya'a, where each study dealt implicitly or explicitly with one aspect of learning. The review will be done while keeping in mind two main issues: characterizing the different aspects of middle school students' learning of mathematics in the mobile phone environment, and examining the factors that influence the different aspects of this learning. The characterization will help mathematics educators be aware of the whole educational scene while teaching with technology, especially mobile devices. This awareness is expected to help teachers prepare for their new roles in a technology-rich environment [27]. On the other hand, examining the factors that influence the different aspects of students' mathematical learning with technology will help mathematics educators plan the activities so that they take care of the different aspects of students' learning. Taking care of these different aspects will facilitate students' learning of mathematics in an environment rich of technology, which will contribute to students' mathematical reflection, problem identification, and decision making [28].

5. Research questions

5.1 What characterizes the different aspects of middle school students' learning of mathematics in the mobile phone environment?

5.2 What influences the different aspects of middle school students' learning of mathematics in the mobile phone environment?

6. Methodology

6.1 Research setting and sample

The experiments of using the mobile phone in teaching and learning mathematics took place in three middle schools in Israel. The experiments in two of the schools were led by the author and his colleague (Dr. Nimer Baya'a from Al-Qasemi Academic College of Education), while the experiment in the third school was led by pre-service teachers (the students of the author) in the frame of their main task in a mathematics didactics course which emphasized the role of technology in mathematics education. The experiments led by the author and Baya'a took place in two middle schools in Israel and extended for 2 years. In the first year, the mathematics in-service teachers in each school selected a group of 25 ninth grade students to participate in the experiment. The selection was made based on students' interest and the ownership of an appropriate cellular phone. The principals of both schools added one weekly lesson with a mathematics teacher for each group, but most of the teaching at these lessons was performed by the researchers and the pre-service teachers. In the second year, one math teacher from each school selected a ninth grade class, and worked with this class during the whole year. Both of the teachers utilized the cellular phone as part of their regular lessons to teach mathematics. They were accompanied by the researchers and the pre-service teachers.

Three pre-service teachers led the experiment in the third school. The pre-service teachers were present in the experiments described above. Doing so, they participated, with their mentor and the Daher, in preparing activities appropriate for learning mathematics with the cellular phone and in directing the students when carrying out the activities. The three pre-service teachers carried out the experiment as their final project in a mathematics didactics course whose instructor was the Daher. At the beginning, the pre-service teachers selected thirty 8 grade students to participate in the project. The selection was done on the basis of volunteering from the side of the students and the ownership of an appropriate cellular phone. All the learning was done by undertaking out-of-class activities that involved exploring the mathematics of real life phenomena. The students utilized the various characteristics and features of the cellular phone to do such exploration. At the beginning, the students carried out activities suggested by the pre-service teachers. Later, in the experiment, when the students had carried out eight real world activities, they started to develop activities themselves. They did that by suggesting real world activities that they judged to be executable with cellular phones. The students usually started from a specific suggestion and developed it further till they considered the activity to be worth carrying out. Overall, the project lasted for twelve weeks.

6.2 The Mathematics Software

The middle school students worked with cellular phone software programs (midlets) which can be downloaded from 'Math4Mobile' site. The Math4Mobile site belongs to the Institute for Alternatives in Education that operates within the Faculty of Education at the University of Haifa [29]. The site includes a pedagogic rationale for using mobile phones in mathematics education, mathematical applications that were developed specifically for mathematics learning, examples of activities that could be used with the applications, and an SMS center that allows forwarding screen content to learning mates. The midlets support the learning of algebra and geometry. In order to perform the activities assigned to them, the students used the algebraic midlets and various tools and technologies embedded in their cellular phones. Mostly, the participants used the midlet "Fit2Go" which enables the user to draw specified points and then to fit a linear Data collections.

Figure 1 describes a quadratic function built with the midlet "Fit2Go" according to three of five points:

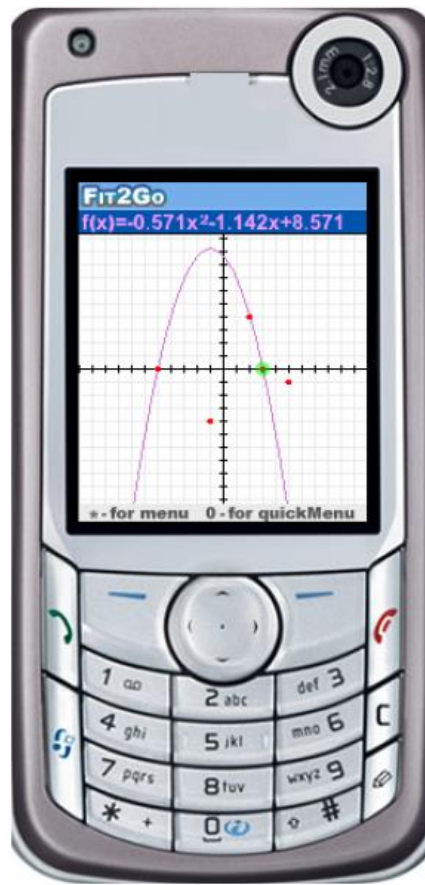


Figure 1: Drawing a parabola in the midlet "Fit2Go" according to three of five points

6.3 The mathematical activities

These activities were described elsewhere (for example [30] and [31]). They involved formal and authentic mathematical activities. One example of the formal activities is finding the influence of the parameters of the linear function on its graph, while one example of the authentic activities is finding the relation between the circumference of a rock and its height, where the students work outdoors on real world rocks to do that.

Following is a lesson plan for the rock activity:

We want to find the rule of the function that describes the relation between the circumference of a rock and its height.

To do so you need to form groups of 4-5 students each, assign roles for the group members, carry out the experiment (outdoors), collect data regarding the circumference of rocks and their heights, register the points on paper and/or in the midlet "Fit2Go", fit a function for these point, and arrive the function rule appropriate for the collected data. Why do you this the function rule is as it is?

6.4 Data collecting tools

To collect data regarding students' learning with the mobile phone, observations of this learning were used in the following studies: [30], [31], [32] and [33], while interviews with students about their learning with the mobile phone were used in all the studies. Other collecting tools were used in some of the studies: writing in an electronic forum [30], filming videos ([31], [32], [33], [34], [35].), writing in a blog ([32], [33], [34], [35]), diaries ([32], [33], [34]), and taking pictures [31]. Below I describe the various collecting tools.

Observations:

We also collected data by observing the students' learning process. We asked the pre-service teachers who conducted the experiment to observe the learning process, take pictures of it, and record it with a video camera. Participating students also took pictures and recorded their activities using their own mobile phones. We collected all the pictures and videos to analyse the students' learning of mathematics.

Filming videos:

Filming videos came to catch the nuances of students' learning, for the observation could not register the whole learning occurrences. All students' learning sessions were videoed, inside as well as outside the classroom.

Taking pictures:

Taking pictures complemented the videoing, where the goal of this collecting was to try to catch specific situations that the video did not concentrate on.

Interviews with the students:

We interviewed students for almost thirty minutes about their experience using mobile phones to learn mathematics, their behaviour and collaboration while performing the mathematical activities, and suggestions for improving the use of mobile phones. The interviews were semi-structured.

Electronic forum:

Students were required to comment on their mathematics learning using mobile phones in a forum hosted on the school's site, to suggest ideas for this use, and to inquire about anything they encounter in the experiment.

Writing in a blog:

To document students' perceptions of their mathematical learning with the mobile phone, we asked them to reflect on their learning, to describe it in terms of their mathematical understanding as well as their emotions towards their learning.

Diaries:

Our pre-service teachers were asked to keep diaries of their experiences, as well as students' experiences in learning mathematics with the mobile phone. The pre-service teachers did that on a weekly basis.

Examples are given below on students' responses using some of the collecting tools.

Videoing

One of the video clips recorded one student telling the other: it's a pity that this is the last activity that we do. I got accustomed to everyone in the group so well, not only the students, the teachers too. They are like family to me. I hope we will get together next year.

Writing in a Blog

The broad question was: Which learning experiences did you have today? What do you think about these experiences?

One student's answer was: Today we did new things. I really feel proud to be part of this project.

Keeping a Diary

One pre-service teacher wrote: The students do everything to make the activities succeed. I think all of us belong to a group living in a new culture in which various methods and tools are used to learn.

Interviews

The question was: what do you think about the project in which you participate?
A student's answer was: this is a novel learning, and I am happy to be part of it.

6.5 Data analysis

All of the studies used qualitative methods to identify the various aspects of students' learning. These methods were general or specific. The general methods were the grounded theory [36] and the discourse analysis [37]. The grounded theory was used to analyze the various aspects of students' learning with the mobile phone ([30], [31], [38], [39]). For example [30] used the grounded theory approach to arrive at a theory regarding students' behavior when learning mathematics with the mobile phone. [34] used discourse analysis and the grounded theory to arrive at students' perceptions of learning mathematics with the mobile phone. Other studies used specific qualitative models to analyze the collected data, for example [32] used knowledge building models to analyze students' building of mathematical knowledge with the mobile phone, while [33] and [35] used models of learning communities to analyze different aspects of students' building of mathematical communities while working with the mobile phone.

7. Findings

The first research question was: What characterizes the different aspects of middle school students' learning of mathematics in the mobile phone environment?

Below is a description of every one of the five aspects of students' learning of mathematics with the mobile phone.

7.1 The cognitive aspect

Different studies out of the eight studies treated the cognitive aspect of students' learning of mathematics in the mobile phone environment. Two studies [31] and [32] treated in detail students' construction of mathematical knowledge when using the mobile phone inside and outside the classroom. Both of the studies described how middle school students used the mobile phone to investigate mathematical relations inside and outside the classroom. The two studies reported students' mathematical investigations, when they worked with various representations of mathematical objects, what enabled them to investigate the objects independently or with the guidance of the teacher.

To describe in more detail the cognitive aspect of the students' mathematical work with the mobile phone, the mathematical software in the mobile phone (the midlets) enabled the students to manipulate the parameters of the algebraic functions, to watch the resulting influence on the graphs and to put conjectures regarding the studied mathematical relations, stating, for example, that a positive 'a' parameter (the slope) implies an increasing function when the function is linear. By the end of their investigation, the middle school students advanced their knowledge in functions and their properties and in mathematical modeling of real life phenomena. [31] added that the intuitive tackling of advanced mathematical objects by the middle school students – enabled by the visual representation and actual realization in real-life situations, prepared the students for the mathematical justification of the relations. Further, [32] described how middle school students carrying out collaboratively outdoor authentic mathematical activities advanced their knowledge of mathematical ideas as a community. Doing so, they demonstrated constructive and critical use of information.

7.2 The meta-cognitive aspect

[32] found that by the end of the experiment, the middle school students advanced their knowledge about mathematical modeling of authentic activities, about the planning and execution of authentic activities, about the design of authentic mathematical activities, and about the analysis of their mathematical actions. Studies relate planning and analyzing one's actions to meta-cognitive actions (for example [40]), so students' advancement of their knowledge can be related to the meta-cognitive aspect of the students' learning.

7.3 The affective aspect

Two issues were studied regarding the affective aspect of learning mathematics with the mobile phone: students' perceptions of their learning with the mobile phone ([34], [38]) and students' emotions that accompanied the learning [39]. [38] found that middle school students were aware of different features of the two technological tools which they used to learn mathematics: mobile phones and web applets. These features were: the availability of the tool, its collaborative feature, its communicational feature, the size of the tool, and the usability of the tool. These features influenced the participants' decisions when, where and how to use each of the tools for the learning of mathematics. More participants preferred the mobile phone over the applet primarily for its small size which makes easy its portability as well as for its communicational facilities. [34] found that students participating in the research perceived various qualities of the mathematics learning that were enabled by the use of mobile phones: (1) exploring mathematics independently (2) learning mathematics through collaboration and team work; where the collaboration is on equal terms (3) learning mathematics in a societal and humanistic environment (4) learning mathematics in authentic real life situations (5) visualizing mathematics and investigating it dynamically (6) carrying out diversified mathematical actions using new and advanced technologies (7) learning mathematics easily and efficiently. In the overall, the students were positively impressed by the potentialities and capabilities of the mobile phones used in the mathematics learning process.

Regarding students' emotions, [39] found that the mobile phone outdoor environment where students work on everyday life activities was appropriate for cultivating mathematics students' positive emotions. The research findings indicated that students had different positive and negative emotions while performing different learning roles when learning with the mobile phone to explore mathematical ideas. For example they had positive or negative emotions depending whether they desired the roles they played or not. Generally the students had more positive emotions than negative ones. These emotions could be related directly or indirectly to the mobile phone environment. For example, students' positive emotions regarding the activity themes could be related directly to the mobile phone environment, because these themes were possible due to the mobility of the mobile phone, as well as the technological tools in the mobile phone: the midlets, the time stopper, etc. These mobility and tools enabled performing activities related to everyday life, related to outdoor phenomena, related to the students themselves or to an issue or a subject that the students liked to do. Students' positive and negative emotions regarding the outer environment conditions (hot, warm, cold, etc.) could be related indirectly to the mobile phone environment, because the outer environment could be a mathematical learning environment due to the mobility of the mobile phone, but they are not connected to the features of the technological tools in the mobile phone.

7.4 The social aspect

The studies that treated this aspect tried to characterize students' community that developed through learning mathematics outdoors with the mobile phone. [33] found that students learning mathematics outdoors with the mobile phone performed learning activities in collaboration and on their own. Doing so, they were aware of their different roles in the learning community and of the mutual relations between them and the pre-service teachers and among themselves. The students also got to know that the pre-service teachers are not the source of knowledge but direct them in their collaborative building of the mathematical knowledge. So, students' collaboration with the directing of the pre-service teachers made the students' learning successful.

[35] found that the mathematical learning community formed had the following characteristics: (1) The members had equal and humanistic relations with each other, pre-service teachers included, where the humanistic relations were represented in the mutual respect between the members; (2) The community members felt they learned in a novel environment; (3) the community cherished the members' mathematical language. In doing so, the mobile phone environment assisted the students in developing a common language which is specific for them, and

thus influencing their identity as a community of learners; (4) The mobile phone learning environment also encouraged mutuality of the community members' relations, autonomy of the members' decisions and learning, and their active and diverse participation in mathematics learning.

7.5 The behavioral aspect

[32] found that middle school students who learned mathematics outdoors with the mobile phone behaved as mathematicians, especially during the second part of the experiment, when they suggested by themselves real world phenomena to explore using the mobile phone. In particular, when discussing the experiments, the students presented supporting evidence about the results they obtained and described the nuances of the activities.

The second research question was: What influenced the different aspects of middle school students' learning of mathematics in the mobile phone environment?

Regarding the factors which influenced students' learning aspects, the findings section will address each aspect and the factors influencing it as in the reviewed papers, but the discussion section will follow a different strategy and discuss the reciprocal influences initiated by the different factors and running through the different learning aspects. The finding section will follow the analytic approach because the reviewed papers treated generally a specific aspect of students' learning of mathematics.

7.6 Cognitive and metacognitive aspects

[31] found that the mobile phone helped students carry out activities involving formal mathematical phenomena and at the same time authentic real-life phenomena which they modeled mathematically. Doing so, they worked individually and collaboratively with diverse, specific and general mathematical concepts and at the same time practiced specific and general mathematical procedures, which expanded their mathematical knowledge and meta-knowledge. This means that the factors influencing the cognitive and meta-cognitive learning of the students were: the features of the mobile phone, the characteristics of the learning activities, and the social context.

7.7 The affective aspect

Here too, two sides of the affective aspect will be considered: students' perceptions of their learning and students' emotions. [34] and [38] found that what influenced the students' perceptions of their learning with the mobile phone were its characteristics, and specifically the general features: enabling visualization of the mathematical phenomenon or problem, enabling the manipulation of the mathematical objects (making the students able to learn the mathematical relations independently), the size of the tool's interface, its portability, and their communicative potentials. These special features could hinder or encourage the use of the devices for learning mathematics. For example, the relatively small screen of the mobile phone hinders its use in general and by students who have sight problems. [34] also pointed out that the novelty of the experiment and the use of mobile phones in mathematics learning were the main characteristics perceived by the students as influencing their decision to join the experiment of using the mobile phone to learn mathematics. In addition, they reported that the students also perceived the qualities of learning enabled by the mobile phones as influencing their learning.

Regarding factors that influence students' emotions, [39] found that the factors which influenced students' emotions most were the activity characteristics (task conditions), the outer environment conditions, and the student's conditions.

7.8 Social aspect

[35] pointed at the main properties of the learning environment that influenced the learning community's characteristics. These were: the context of learning (out-of-class learning and authentic activities), the learning tool (the mobile phone in our case), support (plurality of associations with inner and outer factors: parents, the mathematics teacher, other teachers, etc.), social conditions of learning (equal relations) and the role of the student (autonomy). Further, [38] found that students considered as benefits two characteristics of the mobile phone: (1) the mobile

phone's option to collaborate in solving mathematical problems in the classroom, by working together, usually in pairs using the same phone, and from distance, by sending SMS and MMS messages to each other in order to solve difficult mathematical problems. and (2) learning anytime and not just in the lesson time. So, these two characteristics influenced the social aspect of students' learning with the mobile phone.

7.9 The behavioral aspect

[30] used the grounded theory approach to characterize students' learning of mathematics using the mobile phone. Doing so, they took into considerations the various components of the grounded theory: conditions, actions and interactions and consequences. A conceptual framework was developed out of the relations among the various components. This framework showed that the factors influencing students' behavior in the mobile phone environment were: the principals' and coordinating teachers' involvement, mobile phone features and qualities, the activities themes and requirements, the learning modes (in-class, out-of-class), experiment phase: preparation, activity, after activity.

8. Discussion

The discussion will treat the findings of every one of the tow research questions.

Characterizing middle school students' learning of mathematics in the mobile phone environment

8.1 The cognitive aspect of students' learning

What distinguished middle school students' cognitive experience in the mobile phone environment is the investigative nature of this experience, where the students investigated mathematical ideas and relations on their own using a technological tool. This investigative experience and work of the participating students is similar to students' mathematical work with technology described by other researchers. For instance, the use of spreadsheets to investigate mathematical concepts has been described by [41] for middle school mathematics, by [42] for secondary school mathematics, and by [43] for college mathematics. The difference here is that the students used a more accessible technological tool, and thus the opportunity of its use increases over time. The investigative mathematical work of the students included a sequence of mathematical processes which enabled them to investigate the mathematical relations in authentic outdoor activities: manipulating the parameters of the algebraic functions, watching the resulting influence on the graphs and conjecturing regarding the present mathematical relations present in outdoor real life phenomena. The conjecturing was followed by proving the conjectures, following the patterns arrived at with the help of the midlets. [40], as reported by [44], pointed at the conjecturing process as what mathematicians regard as engaging in doing mathematics. So, this sequence of mathematical processes described above and enabled by the use of the mobile phone, facilitated the cognitive work of the students as mathematicians. Specifically, technology helped students apply mathematics through modeling to their own lives, better understand their world, and thus gain insight into the usefulness of mathematics and its learning [45]. Further, technology was the bridge for the cognitive gap that hinders students from carrying out modeling of mathematical phenomena and tasks [46].

[31] pointed at the intuitive tackling of advanced mathematical objects by the middle school students, when they performed their investigative work, as means for justification of the relations between the objects. This means that the mobile phone environment encouraged an approach described for a long time now as a desired approach for mathematics learning: the intuitive approach ([47], as reported by [48]).

8.2 The meta-cognitive aspect of students' learning

[32] described the advancement of meta-cognitive processes by middle school students, primarily about the design of outdoor activities, the planning of their performance and their actual execution. This advancement can be related to the meta-cognitive aspect of the students' learning [49]. Further, it could be said that these metacognitive processes supported students' cognitive processes described above ([50], [51]).

8.3 The affective aspect of students' learning

It could be said, depending on students' perceptions of their mathematical mobile learning reported by two of the reviewed studies ([34], [38]), that using the mobile phone to teach mathematics, where authentic activities are available for students, can help maintain a humanistic mathematics learning in the sense of Carl Rogers, where Rogers stressed the importance of freedom for students to learn and that students grow and succeed when the teacher has a personal and open relationship with them [52]. In the reviewed studies, students' perception of freedom was due to their ability to plan the carrying out of the mathematical investigation or outdoor activity and due to their ability to suggest the outdoor activities that they would carry out with the help of the mobile phone. The equal relations in the group and with the teacher also contributed to their perception of the humanistic environment in which they learned.

Regarding students' emotions, these emotions were positive in general, so the mobile phone could be used in the mathematics classroom to cultivate students' positive emotions towards mathematics. This makes the use of mobile technologies in learning mathematics one way to change students' view that mathematics is a rigid and dry subject [53].

8.4 The social aspect of students' learning

The mobile phone environment contributed to its members' better preparation for a democratic life, a goal aspired in mathematics education ([54], [55], [56]). The development, nourishment and maturation of the learning community to be similar to a community of experienced applied mathematicians who use mathematical formulae to study everyday phenomena (as reported for example by [33]) emphasizes that the use of the mobile phone in outdoor activities can maintain a collaborative learning of the students, which makes this use aligned with mathematics reform efforts ([56], [57]).

8.5 The behavioral aspect of students' learning

[33] found that students behaved as mathematicians, looking for real life phenomena to investigate mathematically. These findings strengthen the claim of [58] that students have the ability to behave as mathematicians. This claim is especially right when providing students with appropriate tools, the mobile phone in our case, and appropriate mathematical activities, authentic outdoor activities in our case.

The second research question was: What influences middle school students' learning of mathematics in the mobile phone environment?

Looking at the factors that influenced students' learning of mathematics with the mobile phone, we can arrive at common factors that influenced the five aspects of students' learning. These factors are related to: (1) their initial expectations of the technological tool and the activities, (2) their initial perceptions of mathematics, (3) factors associated with the tool, (4) factors associated with the activity, and (5) students' relations with others (associations with others and quality of relations with others: equal relations among students and between students and teacher). Figure 2 describes the overall interactions of the factors and the aspects.

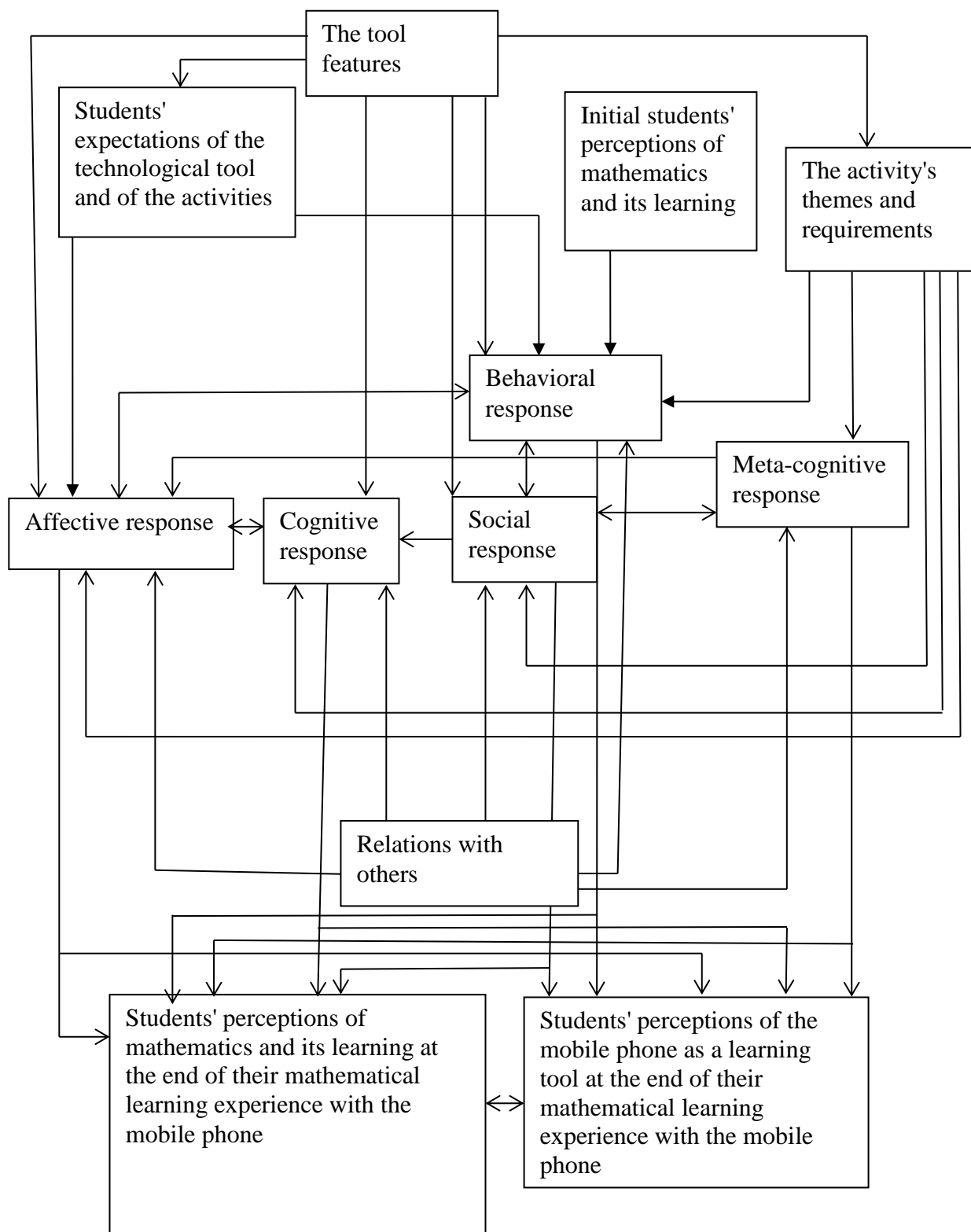


Figure 2: interactions between the factors and aspects of students' learning of mathematics in the mobile phone environment

Two main learning conditions influenced at the beginning the participating students' preparedness to learn mathematics with the mobile phone: students' expectations of the technological tool and of the activities and students' perceptions of mathematics and its learning. In our case, students' expectations of the technological tool and of the activities influenced positively their mathematical learning, and in particular the students who had not had positive image of mathematics. The students, having positive emotions about the mobile phone, transferred these emotions towards their learning of mathematics with the this device, which in turn influenced positively their learning actions, and, in the way, their cognitive responses to the outdoor mathematical activities. The tool also influenced directly and positively students' cognitive responses through two main features of the midlets: enabling different representations of mathematical objects and enabling their manipulation. These two features encouraged the students to make conjectures regarding mathematical relations between the mathematical objects and facilitated proving these relations. The two features also facilitated building mathematical models representing the relations. These influences of the tools' features on the cognitive aspect of students' learning led to influences on the affective aspect of students' learning, because the students had positive emotions and perceptions regarding their ability to see mathematical objects and work independently to discover mathematical relations between them [39]. The positive affective responses led to more engaged learning behaviour of the students [59].

It could be said that the tool also influenced directly the behaviour and the social aspects of students' learning, enabling them to work outdoors and model real life phenomena collaboratively, each time influencing, as a consequence, the other aspects. For example, the tool features (mobility, having multiple devices, and having midlets) enabled preparing activities that probe real life phenomena and could be performed outdoors. These activities influenced students' learning behaviours: investigating real life phenomena outdoors and thus experience mathematics as an applied science. These learning behaviours made the students' enjoy their learning and perceive mathematics as an applied subject. The students' working on real life phenomena (their learning behaviour) also encouraged them to work collaboratively in order to perform all the required learning actions, and, at the same time, it made them perform specific cognitive actions, for example engaging in modelling actions: measuring, assigning points, matching a function, and relating the function to the real life phenomena.

The previous description shows how the tool influences reciprocally the various aspects of students' mathematical learning, and specifically how it can influence directly any one of the learning aspects and as a consequence the other aspects.

The described influence of the tool's features on students' learning is supported in the literature, for example [60] says that tools mediate the learning activities in which students engage. In addition, [61] observes that tool characteristics influence students' learning: "Tools matter: they stand between the user and the phenomenon to be modelled, and shape activity structures" (p. 341).

Regarding the activity, it could be said that the activity themes influenced primarily students' learning behaviour, where students' behaviour outdoors was more similar to applied mathematicians. This happened because the outdoor activities encouraged students to investigate real life phenomena. In addition, students' learning behaviour differed depending on the activity phase, where they had more serious discussion towards the end of the activity. The participants investigated real life phenomena by modelling them mathematically, a cognitive learning action which advanced the mathematical knowledge of students [62]. At the same time, investigating real life phenomena outdoors encouraged students to engage with meta-cognitive actions, as planning and reflective thinking about this planning. All these students' learning experiences influenced students' perceptions of their learning with the mobile phone [34]. Factors associated with the activity (task conditions and the outer environment conditions) also influenced also students' emotions and their social behavior, where the outdoor activities encouraged students, as described above, to work collaboratively due to the various actions required to perform the activity. Further,

working collaboratively made the students enjoy their learning because the collaboration made their learning easier, in addition

The previous influences of the activity's themes, requirement and features are supported in the literature, for example, [63] says that the group activity influences the learning in the group. In addition, [14] depends on the research from the field of computer-supported collaborative learning (CSCL) to conclude that the CSCL task properties determine how students cooperatively work with the computer.

Regarding relations with others, these relations contributed to students' cognitive learning when the students investigated real life phenomena in a learning environment characterized by equal relations between the students and with the teacher. The equal relations encouraged the students to engage in investigating mathematically the real world phenomena and to arrive at appropriate mathematical relations. This is true also regarding students' discussions, where the equal relations encouraged the students to present and defend mathematically their findings and their mathematical models which they arrived at. Relations with others also affected students' meta-cognitive learning, where the equal relations made students practice meta-cognitive acts, like planning the carrying out of the real world activities. The success of these cognitive and meta-cognitive responses encouraged by the equal relations with others helped students develop positive emotions towards their learning of mathematics. The equal relations in the group and the classroom also influence directly and positively students' emotions [34], which in turn made the students more actively carry out the activities (i.e. affected positively their learning behaviour) and try to arrive at mathematical relations more enthusiastically and hardly (i.e. affected positively their cognitive response). This influence of the affective responses on students' behavioural and cognitive responses is supported by [64], as reported by [65], where they described a similar influence of a person's behaviour, but when the person does not have a positive affect for his/her learning. Specifically, they claimed that in this situation, it will be very hard for that person to learn on a cognitive or behavioural level. Other researchers treated the previous relation, for example [66] regards emotions as influencing students' cognitive learning, saying that they "influence our ability to process information and to accurately understand what we encounter" (p. 90). Further, [67] regards emotions as influencing students' learning behaviour, saying that students learn and perform in a more successful way when they have positive emotions (security, happiness, and excitement) about the subject matter. The previous described relations between the different aspects of students' learning make us suggest that these different aspects of learning are related to each other, and that the mobile phone environment, which is one example of the technological learning environment, contributes positively to all of them, on condition that we, as teachers, provide students with appropriate authentic and collaborative activities.

The relations with others also influenced directly the behaviour aspect of students' learning, where students' associations with the researchers, with the principals and the coordinating teachers influenced how the actual learning of the students proceeded [30]. This influence of students' students' behaviour led to positive influence on the other learning aspects of the students.

Other factors which influenced students' learning are related to the students themselves. These were students' conditions (tired) and how they perceived themselves (able to work in the sun, etc.).

9. Conclusions

This review came to describe the whole educational scene of middle school students' learning of mathematics in the mobile phone environment. It did that by reviewing eight already published studies of the author alone and of the author with Baya'a and which generally treated implicitly or explicitly of the following five aspects of students' learning: the cognitive, the meta-cognitive, the affective, the social and the behavioural.

What distinguished middle schools students' cognitive experience in the mobile phone environment is the investigative nature of this experience, where the students investigated

mathematical ideas and relations on their own using a technological tool which they could take outside the classroom to investigate real life phenomena and situations. The investigative students' work was characterized by watching visually the behaviour of mathematical objects, conjecturing regarding the relations between these objects and discussing these relations. On the other hand, it was characterized by building mathematical models of real life phenomena. These characterizations agree with learning experiences pointed at by mathematics education researchers and institutions as supportive for students' learning of mathematics ([56], [68]).

The cognitive experience of the participating students could be enabled due to their meta-cognitive processes which included the planning of teacher-suggested activities, reflecting collaboratively about the mathematical relations arrived at – during discussion, and designing new outdoor activities. This implies the interconnectedness of students' cognitive and meta-cognitive experiences ([23], [49]).

The meta-cognitive experience of the students was done in an environment that the students perceived as supporting their freedom to plan, design and execute. The equal relations in the group and with the teacher also contributed to their perception of the humanistic environment in which they learned. Further, students' emotions while learning mathematics with the mobile phone were positive in general, which could be explained by their positive perception of their learning and due to the social options of this learning, where the activities given and suggested by the students encouraged them to work collaboratively, maintaining a supportive community of learning that behaved as a community of experienced applied mathematicians.

9. References

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